Determination of modulus of elasticity for glass fibre reinforced polymers

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DETERMINATION OF MODULUS OF ELASTICITY FOR GLASS FIBRE REINFORCED POLYMERS

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Preliminary notes

The subject of this paper is to determine the idealised stress-strain diagram (primal curve) as well as a numerical expression of a straight line and a parabolic curve for glass fibre reinforced polymers (GFRP). The intention is to create a tool, by which one could define the stress-strain diagram for any given glass fibre based on tensile strength. GFRP is a material that is increasingly being used in structures. Because of that we need to know some of its basic characteristics or of other similar composite materials, so as to be able to calculate these structures. This paper presents the modulus of elasticity determined by testing glass fibre reinforced polymer samples and a proposition of an idealised stress-strain diagram as well as a numerical determination of the modulus.

Keywords: composite, elastoplastic material, glass fibre reinforced polymer (GFRP), modulus of elasticity, primal curve, stress-strain diagram

Određivanje modula elastičnosti stakloplastike

Prethodno priopćenje

Predmet ovog rada je određivanje idealiziranog dijagrama naprezanje-deformacija (primarna krivulja) te brojčana definicija pravca i krivulje drugog reda za stakloplastiku (GFRP). Namjera je, također, da ovaj rad postane alat kojim bi se mogao definirati dijagram naprezanje-deformacija za bilo koju drugu stakloplastiku na osnovu rastezne čvrstoće. Stakloplastika je materijal koji je ušao u širu primjenu u graditeljstvu, što zahtjeva i poznavanje određenih karakteristika same stakloplastike ili sličnih kompozitnih materijala, kako bismo mogli proračunati te konstrukcije. U ovom radu dan je modul elastičnosti za stakloplastiku određen ispitivanjima te prijedlog idealiziranog radnog dijagrama stakloplastike i računskog određivanja modula elastičnosti.

Ključne riječi: dijagram naprezanje-deformacija, elastoplastični materijal, kompozit, modul elastičnosti, primarna krivulja, stakloplastika (GFRP)

1
Introduction
Uvod

The progress in civil engineering has caused a wider use of modern composite materials, as it has already been the case with reinforced concrete. Composites used today are normally made of a matrix and some kind of filler that reinforces the material. The filler or the reinforcement is usually a light, strong and brittle material and the filler is mostly a material with more elastic characteristics, whose function is to bond, to transfer the loads and to protect the filler. The application and analysis of composite materials was the subject of the papers by many authors [1, 4, 5], as well as by authors of books and manuals [2, 3].

The basic characteristic of a material, seen through the Hooke's law, is the modulus of elasticity, known as the Young's modulus. To put it shortly, it represents the ratio of stress and strain ($E = \sigma/\varepsilon$). This value is defined in the elastic area. For materials that have a distinct plastic area it is usual for the value to be known only in the elastic area. By using an idealised stress-strain diagram, which is normally defined by straight lines, the elastic area where values of the modulus are exact and the area of plastic behaviour where the value of the modulus gradually decreases, is described. All materials have both elastic and plastic area. In order to simplify any calculations, one value of the modulus is used, and for the plastic area it is normally defined to begin when the irreversible deformations reach the value of, for example 0,1 %.

The modulus of elasticity is considered to be the basic characteristic of a material and it is used for most calculations of any structures, so it is important to know its value. Composite materials have a long history of application in civil engineering and in recent history we have been using composites like carbon, teflon, glass fibre reinforced polymers, concrete reinforced with carbon

fibres, rebar and similar. Because of the constant use of new materials there is a need to define some basic characteristics of a material through, for example, tensile strength.

2
The testing of samples
Ispitivanje uzoraka

The data from the testing of samples is taken from the experiments of glass fibre reinforced polymers from the literature [1], for two similar samples broken with tensile force. The samples showed elastic behaviour up to one third of the breaking value, and on the remaining two thirds the material behaved plastically. That is, for every increase of the force the value of the modulus decreased slightly. And after the release of the force and the reapplying, the plastic deformation started to reoccur with the increase of the force in comparison with the previous value. In total, two samples were observed and they showed similar shapes of the stress-strain diagrams and modulus of elasticity.

The shapes of the samples were taken according to the American Standard ASTM D 3090, as shown in Fig. 1. The machine used for the testing is described in more detail in the literature [1], and is shown in Fig. 2.
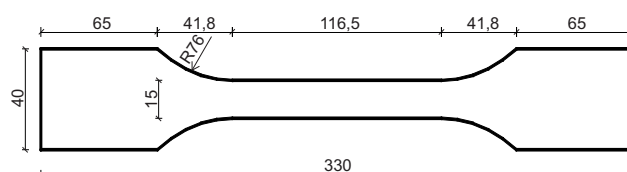
Figure 1 *Geometry of the tested GFRP sample (units in mm)*
Slika 1. *Geometrija ispitivanog GFRP uzorka (jedinice u mm)*

Figure 2 Testing machine with the samples
Slika 2. Ispitni stroj s uzorcima

3

The determination of the modulus of elasticity for glass fibre reinforced polymer
Određivanje modula elastičnosti stakloplastike

The data from the testing have shown that glass fibre reinforced polymers behave as an elastic material up to a certain value of stress, and after that they start to deform plastically all the way to the breaking point. The characteristic of this material is that after every increase of strain, that is, after every increment of the testing procedure, the plastic deformation begins at the value at which the previous increment finished. Knowing this, it is possible to divide the stress-strain diagram into two areas. The elastic area expands up to one third of the bearing value. In this area the material does not exhibit irreversible deformations, that is, it behaves as an elastic material (Fig-s 3 and 5). After exceeding this value the material starts to show irreversible deformations. The line that describes the relaxing of the samples appears to be fairly linear, which implies the assumption that the material does not need time to take the original shape, at least from the elastic area (Fig-s 4 and 6). It is also visible that the ultimate strength of these samples is roughly 60-65 N/mm^2, at which time the material exhibits strain in the value of about 0,2 %.
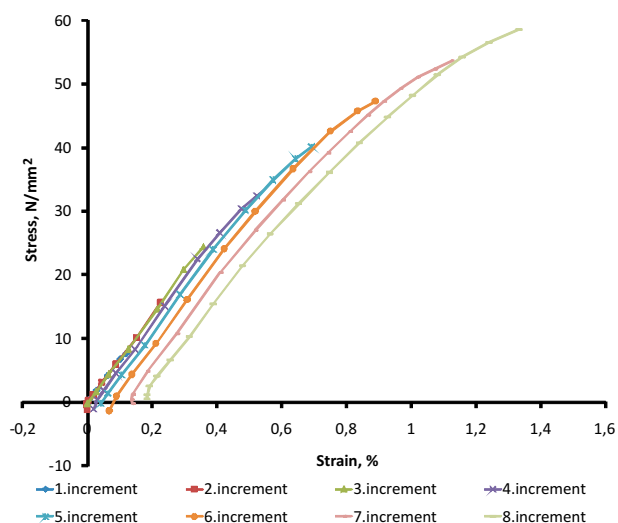
The mean value of the modulus of elasticity for the observed samples is 66,4 N/mm^2, for the elastic area, and it falls to around 20 N/mm^2 prior to the breaking point. If we construct a diagram using the data for the elastic area, as a line, which would represent the behaviour of these samples, then for the sample 1 it would look as shown in Fig. 7 (for better appearance additional 3,5 N/mm^2 were added on the stress axis), and the mathematical expression is also shown in Fig. 7. Furthermore, this was also done for the plastic area of all the increments, together with the mathematical expression, and shown in Fig. 7 (also with additional 3,5 N/mm^2 on stress axis). One irregularity is visible in Fig. 5, in the shape of the line deflection at the strain value of 0,2 % and stress value of about 15 N/mm^2. This can be attributed to the slipping of the samples at the attachment points. In Fig. 8, which represents the second sample, additional 3,5 N/mm^2 was also added on the stress axis for better visibility.
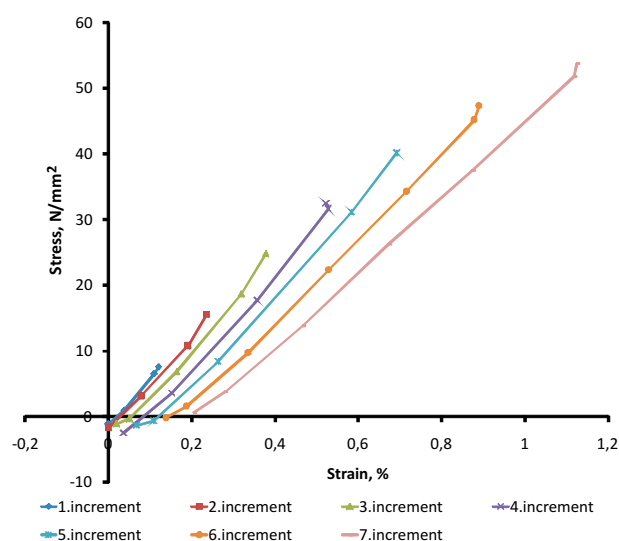

Figure 4 σ-ε diagram, sample 1, 7 increments, releasing stress
Slika 4. σ-ε dijagram, uzorak 1, 7 priraštaja, oslobađanjem naprezanja

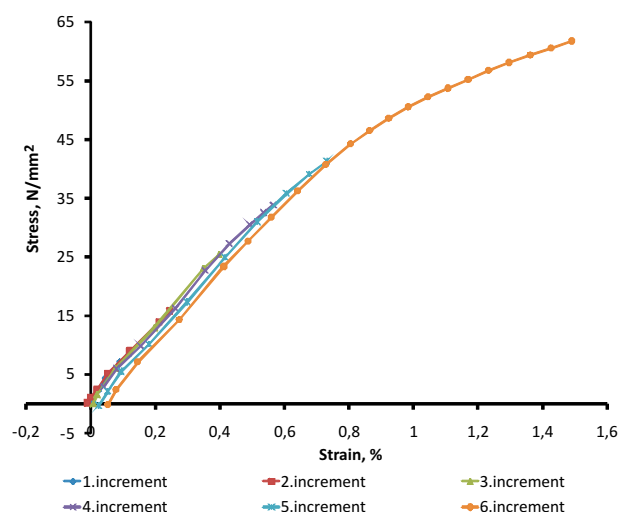*Figure 5 σ-ε diagram, sample 2, 6 increments, applying stress*
Slika 5. σ-ε dijagram, uzorak 2, 8 priraštaja, primjenom naprezanja

It is visible that the σ-ε diagram for fibre reinforced polymer consists of a straight line, which represents the elastic area, where the stress and strain have a linear relation, and that this area reaches up to 1/3 of the breaking stress. After that, 1/3 of the area follows the 2/3 plastic area, which is fairly approximated by a second order curve.
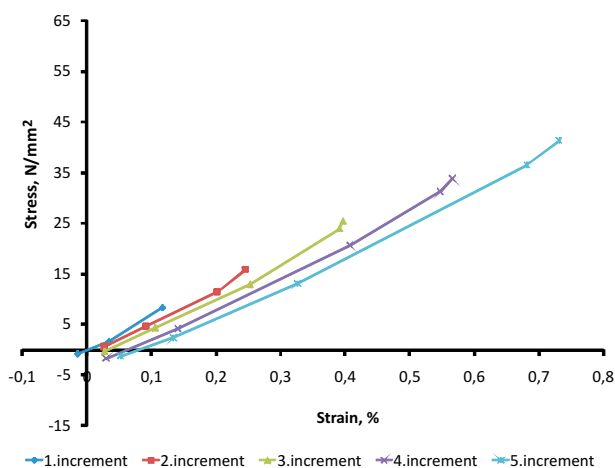

Figure 3 σ-ε diagram, sample 1, 8 increments, applying stress
Slika 3. σ-ε dijagram, uzorak 1, 8 priraštaja, primjenom naprezanja

Figure 6 σ-ε *diagram, sample 2, 5 increments, releasing stress*
Slika 3. σ-ε *dijagram, uzorak 2, 5 priraštaja, oslobađanjem naprezanja*
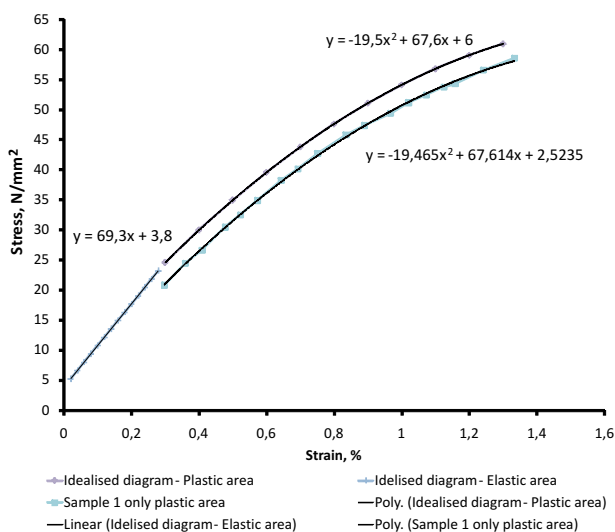
Figure 7 *Idealised σ-ε diagram – sample 1*
Slika 7. *Idealizirani σ-ε dijagram – uzorak 1*
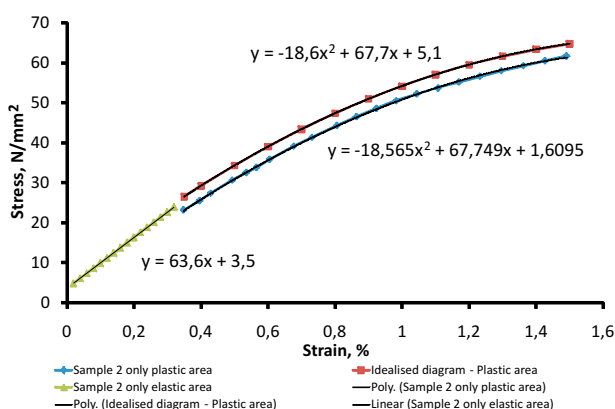
Figure 8 *Idealised σ-ε diagram – sample 2*
Slika 8. *Idealizirani σ-ε dijagram – uzorak 2*

The determination of the modulus of elasticity in the area of 1/3 breaking stress is simple and can be seen from Fig-s 7 and 8. If there is a need to know the value of the elasticity modulus for a specific stress of a material, (for example the stress at which the material already suffered some plastic deformation), this could be done by solving the second order equation, that also describes the diagram curve in Fig-s 7 and 8.

$$y = -19,5 \cdot x^2 + 67,6 \cdot x + 2,5 \qquad (1)$$

$$E = \frac{\Delta\sigma}{\Delta\varepsilon} = \frac{\sigma_2 - \sigma_1}{\varepsilon_2 - \varepsilon_1}. \qquad (2)$$

If we assume that σ_2 is for a small portion larger than σ_1, or in other words that $\Delta\sigma$ is 0,1 N/mm^2, the strain can be calculated by solving the equation (3), so the following can be written:

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4a \cdot c}}{2a} \qquad (3)$$

$$\varepsilon_{1,2} = \frac{-(-67,6) \pm \sqrt{(-67,6)^2 - 4 \cdot 19,5 \cdot (\sigma_2 - 2,5)}}{2 \cdot 19,5} \qquad (4)$$

$$\varepsilon_{1,2} = \frac{-(-67,6) \pm \sqrt{(-67,6)^2 - 4 \cdot 19,5 \cdot (\sigma_2 - 0,1 - 2,5)}}{2 \cdot 19,5}. \qquad (5)$$

Combining equation (2) with equations (4) and (5) we obtain next formula:

$$E = \frac{0,1}{\frac{-(-67,6) \pm \sqrt{(-67,6)^2 - 4 \cdot 19,5 \cdot (\sigma_2 - c)}}{2a} - \frac{-(-67,6) \pm \sqrt{(-67,6)^2 - 4 \cdot 19,5 \cdot (\sigma_2 - 0,1 - c)}}{2a}} \qquad (6)$$

from which we can define the modulus of elasticity for any specific stress in the plastic area of glass fibre reinforced polymers. As the correct solution for strains $\varepsilon_{1,1}$, $\varepsilon_{1,2}$, $\varepsilon_{2,1}$ and $\varepsilon_{2,2}$, for any strain the positive result is chosen. Shown in Tab. 1 is a significantly different modulus of elasticity calculated for several stresses in the plastic area.

Table 1 *Calculation of several modules of elasticity for different stresses in the plastic area for sample 1*
Tablica 1. *Proračun nekoliko modula elastičnosti za različita naprezanja u području plastičnosti za uzorak 1*

Δσ N/mm^2	y = ax^2 + bx + c - ax^2- bx - (c + y) = 0			σ N/mm^2	35	40	45	50	55	60
	a	b	c	E N/mm^2						
0,1	19,5	-67,6	-2,5		45,2	40,6	35,5	29,5	21,9	9,4

Figure 9 *Idealised bilinear σ-ε diagram*
Slika 9. *Idealizirani bilinearni σ-ε dijagram*

In order to simplify even more the idealised stress-strain diagram, it is possible to present it as a diagram consisting of two linear lines, i.e. bilinear diagram, as shown in Fig. 9.

When looking at the bilinear approximation it must be considered that the value of modulus of elasticity is also constant in the plastic area, while it is visible that after 2/3 of

breaking force (or stress) the value of modulus falls under the approximated value. In other words, this kind of a diagram understates the value of E from 1/3 to 2/3, and overstates the value of E above the 2/3 of the breaking force.

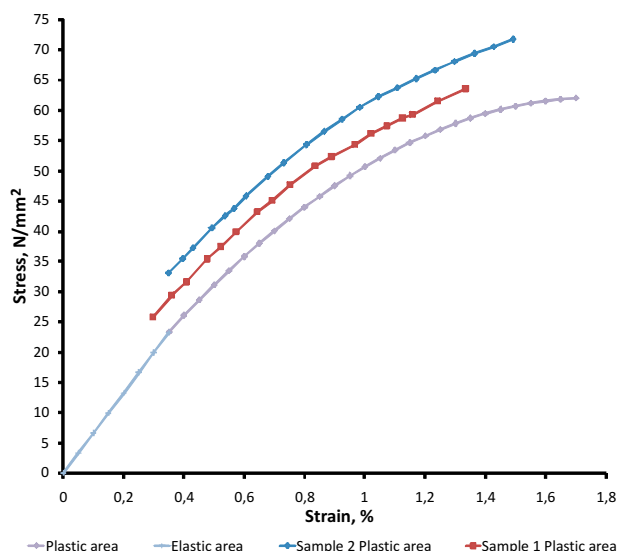According to these statements, the use of linear - parabolic idealised diagram shown in Fig. 10 is recommended.

Figure 10 *Comparison of different σ-ε diagrams*
Slika 10. *Usporedba različitih σ-ε dijagrama*

If the mean value of the two samples is considered, it can be said that the behaviour of glass fibre reinforced polymers in the elastic area or the value of the modulus of elasticity can be described by the following equation:

$$y = 66,4 \cdot x. \tag{7}$$

The plastic area is approximated by a curve of the second order and if the mean value of two samples is considered, it can be said that the behaviour of glass fibre reinforced polymer in the plastic area or the value of the modulus of elasticity can be described by the following equation:

$$y = -19,05 \cdot x^2 + 67,65 \cdot x + 2,05. \tag{8}$$

4
Conclusion
Zaključak

It has been shown that on the basis of tensile strength of glass fibre reinforced polymers the value of the modulus of elasticity can be determined for any area (elastic or plastic) the material is in. The division is made in elastic and plastic area. Elastic area is approximated by a straight line and it is situated in the first third of the bearing strength of the material. The remaining area, until the breaking point of the material, is the plastic area.

According to the results we recommend the use of the linear - parabolic shape of the idealised stress-strain diagram, whose shape describes the testing results very well, as shown in Fig. 10. In addition to that, there is an obligatory division of the stress-strain diagram in elastic area, which goes till 1/3 of the tensile strength, and plastic area, which goes from 1/3 of tensile strength to the breaking point of the material.

5
References
Literatura

[1] Štefić, T.; Jurić, A.; Maričić, S. Strength of glass fiber reinforced polymers tested using strain gauges and displacement meters (LVDT-S) (Nosivost uzoraka od stakloplastike ispitivana tenzometrima i mjeračima pomaka (LVDT-ima)). // Treći internacionalni naučno-stručni skup Građevinarstvo-nauka i praksa (2010), 759-764 (in Croatian).
[2] Šimunić, Ž. Polymers in Civil Engineering (Polimeri u graditeljstvu), University of Zagreb, Faculty of Civil Engineering, Zagreb, 2006. (in Croatian).
[3] Gerdeen, J. C.; Lord, H. W.; Rorrer, R. A. Engineering Design with Polymers and Composites. // CRC Press, Taylor & Fancis Group, Boca Raton, 2006.
[4] Lacković, V.; Šimić, V. Behaviour of composite materials subjected to combined load (Ponašanje kompozitnih materijala pri složenom opterećenju). // Građevinar, 58, 7(2006), 549-557 (in Croatian).
[5] Hollaway, L.C. The evolution of and the way forward for advanced polymer composites in the civil infrastructure. // Construction and Building Materials 17 (2003), 365–378.

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